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Dynamics of bond and stock returns☆

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ABSTRACT

A production-based equilibrium model jointly prices bond and stock returns and produces time-varying correlation between stock and real treasury returns that changes in both magnitude and sign. The term premium is time-varying and changes sign. The model incorporates time-varying risk aversion and two physical technologies with different cash-flow risks. Bonds hedge risk-aversion shocks and command negative term premium through this channel. Cash-flow shocks produce co-movement of bond and stock returns and positive term premium. Relative strength of these two mechanisms varies over time. The correlation is a powerful predictor of relative bond-stock and long-short equity returns in the data.

1. Introduction

During the 2008 financial crisis US treasury bonds proved to be excellent hedges against stock market risk. Frequent "flight-to-safety" episodes pushed stock prices and bond yields down simultaneously. Bonds were thus particularly valuable and, according to standard asset pricing theory, were likely to command low or even negative term premium. In the 1990s, however, bond and stock prices tended to co-move, making bonds risky and therefore term premia were likely positive. Fig. 1 shows that correlation of stock returns with *nominal and real* long-term bond (TIPS) returns is variable and changes sign. No existing general equilibrium model, to my knowledge, is able to obtain this result through a purely "real" channel.[1] In fact, in most of consumption-based, habit,[2] and long-run-risk model calibrations, the correlation between real bond and stock returns and real bond term premium are always negative and the real yield curve is always downward-sloping. In this

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[1] Rudebusch and Swanson (2008), David and Veronesi (2013), Bansal and Shaliastovich (2012) specify models in which correlation between stock and *nominal* bond returns can change sign. Jermann (2013) obtain a similar results in a production model with two sectors but without an explicit preference structure.

[2] Bekaert et al. (2010) and Wachter (2006) are exceptions. In these models consumption-smoothing effect of habits dominate the precautionary-savings effect and results in an upward-sloping yield curve and positive real bond risk premium. On the downside, however, these models counter-factually imply that interest rates are high when surplus-consumption ratio is high (in recessions) and low otherwise.

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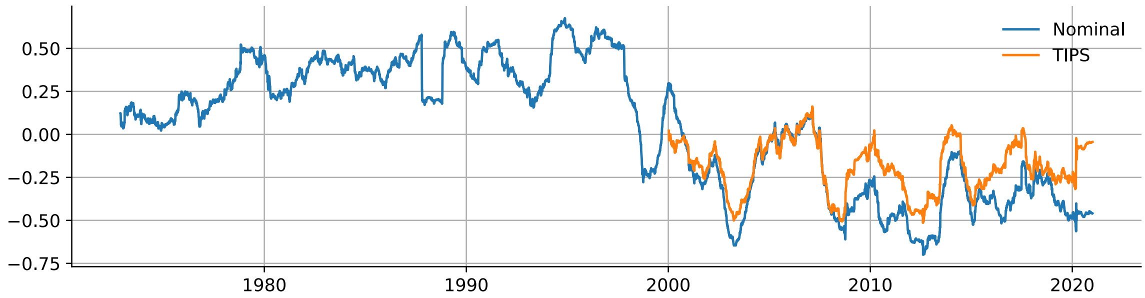

Fig. 1. Rolling correlation between daily equity market and 10-year treasury excess returns. The longer time series (blue) nominal treasuries. The shorter time series (orange) TIPS.

paper I present a production-based general equilibrium model with explicit preference structure that produces time-varying correlation between stock and real bond returns and real term premium that change in both magnitude and sign.

The model features two physical technologies with different amounts of capital risk and adjustment costs to investment. Both technologies produce the same good, but differ in their risk. One technology is more productive, but also has higher exposure to the capital "obsolescence" shock. Using two technologies is a theoretical device to model a continuum of capital of various degree of riskiness across all firms in the economy. The two technologies are designed to model cross-sectional heterogeneity in cash-flow risk *across* firms (e.g., low-risk "utility" companies vs. high-risk innovative "hi-tech" companies) or *within* firms (across firm's projects). When one technology has a good shock (more capital; constant productivity), investors want to rebalance towards the other technology to diversify their capital investment. However, they face adjustment costs, which drives up the price of the technology with no shock. This mechanism, *technology diversification*, produces primarily positive correlation between returns on the two technologies, even when their cash flows are independent. Technology diversification also produces positive correlation between returns on real bonds and stocks in the model. The mechanism is similar to "two trees" in Cochrane et al. (2008) or two technologies in Wang and Eberly (2012) and operates through discount-rate effects induced by general equilibrium market clearing.

The model also features changes in risk attitudes, generated by a time-varying risk-aversion coefficient within Epstein-Zin preferences or, equivalently, by time-varying uncertainty about model misspecification as in Drechsler (2013). When risk aversion rises, investors want to move from riskier to less risky assets, a *flight-to-safety* effect, which produces a negative correlation between returns on real bonds and stocks.

Technology diversification and flight-to-safety together result in time-varying correlation between returns on financial assets with different amounts of cash-flow risk even when their cash flows are independent. When applied to price bonds and stocks, the model produces time-varying correlation between stock and default-free real bond returns, that changes sign. Further, the model produces a time-varying real term premium, which also changes sign. When risk aversion is high, the flight-to-safety mechanism dominates and the return correlation is negative. When risk aversion is low, the technology diversification mechanism results in a positive correlation between bond and stock returns and a positive term premium. Technology diversification relies on slow physical capital reallocation and thus drives low frequency dynamics. Flight-to-safety operates at a higher frequency and is mostly responsible for variation in financial variables in the model.

The structure of the production-based model allows us to learn about properties of the implied consumption growth process that can be used in an endowment economy to replicate the results of the paper. In the model, consumption growth is positively autocorrelated, consistent with empirical evidence. However, the covariance of contemporaneous consumption growth with the infinite sum of all future expected consumption growth is negative. This property, coupled with Epstein-Zin preferences, tends to generate a positive component to real bond term premia and the correlation between returns on real bonds and stocks.

To study the model's predictions, I calibrate the model and find the ICAPM representation of expected returns. This allows me to learn what fraction of the risk premium on any asset comes from technology diversification and flight-to-safety mechanisms. The contribution of technology diversification is positive for all assets, resulting in a "common factor" in returns. The contribution of the flight-to-safety mechanism is negative for bonds and positive for stocks. Bonds therefore hedge some stock market risk because they pay well when market discount rates (risk aversion) increase. The relative magnitudes of the two components change over time across all assets. When discount rates are high, flight-to-safety dominates and we see high stock risk premium, negative bond risk premium, and negative return correlations. The opposite holds when discount rates are low.

Lastly, I show that the model produces realistic dynamics of term structures of yields and term premia, and verify a number of novel empirical predictions implied by the model: (i) Bond-Stock correlation co-moves with empirical measures of risk aversion such as volatility, VIX, and credit spreads, (ii) Bond-Stock correlation has a strong positive effect on the level and a mild negative effect on the slope of the yield curve, and (iii) Bond-Stock correlation is a powerful predictor of bond and equity risk premia, and especially their long-short portfolio, as well as some equity-only long-short portfolios sorted on measures of their bond-likeness.

Related literature

The paper builds on several strands of existing literature. First, many papers analyze bond and stock risk premia separately. Others find evidence linking both markets. Fama and French (1993) note that the term spread predicts the stock market returns. Similarly, Cochrane and Piazzesi (2005) find that a linear combination of forward rates, the “CP factor”, is a good forecaster of government bond risk premia in the cross-section and time-series, and also forecasts stock excess returns. Van Binsbergen et al. (2012) build a DSGE model to jointly price bonds and stocks. Kozak and Santosh (2020) price a cross-section of bond and stock returns within a three-factor empirical ICAPM specification. The last two papers also show decompositions of bond risk premia by maturity.

The paper also relates to the literature that studies the correlation of bond and stock returns. Campbell et al. (2017) explicitly embed time-varying bond-stock covariance in a reduced-form model to explore the changes in risk of nominal government bonds over time. Campbell et al. (2020) study bond-stock correlation in a New Keynesian model with inflation and output gap. Baele et al. (2010) design a dynamic factor model to analyze economic sources of bond-stock comovement. Connolly et al. (2005) find a negative relation between implied volatility and bond-stock correlation, which they attribute to flight-to-quality. Finally, Campbell et al. (2017) describe a puzzle: “some papers have also modeled stock and bond prices jointly, but no existing models allow bond-stock covariances to change signs”. In my model, bond-stock covariances change signs endogenously, due to changing relative strength of two mechanisms, rebalancing and flight-to-safety.

Many general equilibrium macroeconomic models produce a negative real bond risk premium and a negatively sloped real yield curve (and a negative correlation of bond and stock returns). Some exceptions, such as Bekaert et al. (2010) and (Wachter, 2006), produce a positively sloped yield curve within an external habit model by making the consumption smoothing motive dominate the precautionary savings motive. These papers, however, imply that the real short rate is high when the consumption-surplus ratio is low. Others (David and Veronesi, 2013; Rudebusch and Swanson, 2008; 2012) have tried to use the dynamics of inflation to reconcile the failure of standard models to produce a positive term premium. Rich dynamics of inflation within a classical macroeconomic model that implies a negative real term premium can potentially resolve some of the puzzles for *nominal* bonds. Although inflation is an important determinant of prices of nominal bonds, I focus exclusively on *real* bond prices in this paper. Empirical evidence suggests the salient features of the dynamics of real and nominal bond prices are similar.

On a more technical note, the two main mechanisms in this paper heavily depend on several ingredients in the literature. First, the rebalancing mechanism, borrows a lot from Cochrane et al. (2008), who show that having two technologies with infinite adjustment costs produces a comovement, or a “common factor” in returns on assets with independent cash flows in general equilibrium. Martin (2017) generalized this setup to multiple “trees” and power utility. My paper differs from these by endogenizing the size of “trees” and modeling technologies with different amounts of cash-flow risk. The latter feature leads to an endogenous time-varying quantity of risk in the economy, which is essential for producing a sizable positive bond risk premia and a positive correlation of bond and stock returns. Wang and Eberly (2012) while also endogenize the size of “trees”, do not allow for this endogenous risk-taking. Nevertheless, I borrow a lot from Wang and Eberly (2012) in terms of modeling two technologies and using similar analytically tractable functional forms. None of the papers mentioned also allowed for an exogenous variation in risk aversion.

Second, the flight-to-safety mechanism relies on the time variation in risk aversion. The general idea goes back to Constantinides (1990) and Campbell and Cochrane (1999), who studied models with internal and external habits, capable of generating endogenous fluctuations in the curvature of the value function. In these papers, fluctuations in the curvature come purely from the past consumption dynamics. Alternatively, Gârleanu and Panageas (2015) show time-varying risk aversion naturally arises in a general equilibrium with heterogeneous agents. Menzly et al. (2004) specify a process for the consumption-surplus ratio directly, which simplifies the modeling. Bekaert et al. (2010) goes further by allowing the curvature of the value function depend also on exogenous shocks, producing essentially exogenous variation in risk aversion. Finally, Dew-Becker (2014) specify an exogenous process for risk aversion directly, and is the closest setup to the way I model the flight-to-safety mechanism in my paper.

A closely related strand of the literature explores term premium in production economies. Jermann (1998) argues that both capital adjustment costs and habits are needed to replicate the basic business cycle facts and the historical equity premium. The author notes, however, that a tight link between risk aversion and elasticity of intertemporal substitution results in an overly volatile risk-free rate. My model does not suffer from this issue because it employs (Epstein and Zin, 1989) preferences. Jermann (2010) further investigates the role of adjustment costs in generating equity and term premium by examining the role of producers' first-order conditions without an explicit preference structure. Jermann (2013) builds on Jermann (2010) and shows a model that can quantitatively match first and second moments of the real returns on stocks and real bonds. Importantly, the author finds that the model endogenously produces time-varying bond risk premia which can be either positive or negative, as well that bond-stock covariance can change signs endogenously. My paper extends this work by introducing an explicit preference structure to the model.

2. The model

This section starts with introducing the setup and then proceeds to explaining pricing implications and the mechanisms.

2.1. Setup

I now explain the main ingredients of the model.

2.1.1. Production

There are two technologies indexed by $n = \{0, 1\}$. Production function takes a simple AK form: $Y_{n,t} = A_n K_{n,t}$, where $Y_{n,t}$ is the total output, A_n is a constant productivity multiplier, and $K_{n,t}$ is the equilibrium capital of technology n.

A process for evolution of capital features adjustment costs and is given by

$$dK_{n,t} = \phi_n(i_{n,t})K_{n,t}dt + K_{n,t}\sigma_{K,n}dZ_{K,t}, \tag{1}$$

where $i_{n,t} = \frac{I_{n,t}}{K_{n,t}}$ is the investment-capital ratio of each of two technologies, $\phi_n(i_{n,t}) \times K_{n,t}$ is a concave in $i_{n,t}$ installation function (so that an adjustment-cost function is convex), which is homogeneous of degree one in capital, $Z_{K,t}$ is a capital shock, and $\sigma_{K,n}$ is a constant loading of technology n on the shock. I thus assume the capital shock is the same for two technologies, which leads to a single source of cash-flow news in the model. The presence of the shock in the capital-accumulation process of an *AK*-technology is identical to shocks to production possibilities in Cox et al. (1985). These shocks can be interpreted as "obsolescence" shocks: bad realization of a shock leads to the lower level of capital with no change in future productivity.[3]

Each unit of investment increases the capital stock by $\phi_n'(i_n)$ and is valued at q_n, the Tobin's (marginal) q. Competitive firms therefore optimally choose to equate $\phi_n'(i_n) \times q_n$ to unity, the cost of investment. The Tobin's q is hence given by $q_n = Q_n = \frac{1}{\phi_n'(i_n)}$. See Supplementary Materials: Appendix for a more formal argument.

For expositional purposes I assume that technology indexed by $n = 0$ has no cash-flow risk exposure in the rest of the paper. This assumption does not change the main mechanisms of the model but simplifies algebra and intuition significantly and amplifies the technology-diversification effect.

Definition 2.1. The *low-risk technology* has no risk in its capital accumulation process, $\sigma_{K,0} = 0$.

Although the capital-accumulation process contains no risk, returns on the low-risk technology are not instantaneously risk free and are exposed to the discount-rate risk due to the built-in adjustment costs. With adjustment costs, the price of installed capital (Tobin's q) changes over time, affecting the overall value of the technology.

Definition 2.2. The *high-risk technology* has a non-zero loading on the capital shock, $\sigma_{K,1} \equiv \varsigma_K \neq 0$.

In principle, both technologies can possess capital risk. None of the mechanisms of the model requires one technology to have $\sigma_K = 0$. Even in the case when the two technologies are symmetric ex ante, as in Cochrane et al. (2008) and Martin (2013), the capital reallocation mechanism is still operational. There are several reasons for differentiating technologies in terms of their productivity and volatility. First, quantitatively, the magnitude of the capital reallocation mechanism is subtantially smaller when technologies are identical. Second, for expositional purposes it is convenient to treat the high-risk technology as "stock-like" and the low-risk technology as "bond-like," for reasons I will explain in Section 2.3. I will also verify that the two technologies indeed are quantitatively good approximation to stocks and long-term bonds priced within the model in Section 2.2.

Finally, suppose there are two types of competitive firms and that each type can invest in a single type of capital. Firms choose investment to maximize their value,

$$P_{n,t} \equiv p_{n,t} \times K_{n,t} = \sup_{\{i_{n,t}\}} \mathbb{E}_t \int_t^\infty \frac{\Lambda_{t+\tau}}{\Lambda_t}[A_n K_{n,t+\tau} - i_{n,t+\tau}K_{n,t+\tau}]d\tau, \tag{2}$$

where Λ_t is a stochastic discount factor (SDF) that is determined in equilibrium.

2.1.2. Preferences

I specify a stochastic differential utility of Duffie and Epstein (1992b). This utility is a continuous-time version of Epstein-Zin discrete-time specification. Following Duffie and Epstein (1992a) I define a stochastic differential utility by two primitive functions, $f(C_t, J_t) : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}$ and $A(J_t) : \mathbb{R} \to \mathbb{R}$. For a given consumption process C, the utility process J is a unique Ito process that satisfies a stochastic differential equation,

$$dJ_t = \left[-f(C_t, J_t) - \frac{1}{2}A(J_t)\|\boldsymbol{\sigma_{J,t}}\|^2\right]dt + \boldsymbol{\sigma_{J,t}}d\boldsymbol{Z_t}, \tag{3}$$

where $\boldsymbol{\sigma_{J,t}}$ is an $\mathbb{R}^2$-valued square-integrable utility-"volatility" process, J_t is a continuation utility for C at time t, conditional on current information, $f(C_t, J_t)$ is the flow utility, $A(J_t)$ is a variance multiplier that penalizes the variance of the utility

[3] The "Internet Crisis" of 2002 could be used as an example. During the crisis we learnt that the value of existing "internet" capital is lower than it was previously hypothesized; yet, expectations of future innovativeness and growth of the technology sector as a whole might have stayed constant.

"volatility" $\left\|\boldsymbol{\sigma_{J,t}}\right\|$, and $\boldsymbol{Z_t} \equiv (Z_K, Z_\alpha)^\top$ is a vector of shocks. A pair (f, A) is called an aggregator. I use a Kreps-Porteus (Epstein-Zin-Weil) aggregator, defined as

$$f(C, J) = \frac{\delta}{\rho}\frac{C^\rho - J^\rho}{J^{\rho-1}} = \frac{\delta}{\rho}J\left[\left(\frac{C}{J}\right)^\rho - 1\right] \tag{4}$$

$$A(J) = -\frac{\alpha}{J}, \tag{5}$$

where $\rho = 1 - \frac{1}{\psi}$ and ψ is the elasticity of intertemporal substitution; δ is a subjective discount factor, and α is the risk-aversion parameter. Duffie and Epstein (1992a,b) show finding an ordinally equivalent aggregator $\left(\bar{f}, \bar{A}\right)$ is possible such that $\bar{A} = 0$, a *normalized* aggregator. Most papers that employ Epstein-Zin-Weil preferences use this type of aggregator.

I take a different approach and use an *unnormalized* aggregator as defined explicitly in Eqs. (4) and (5). Although such a representation requires computing an additional variance term in Eq. (3), it allows me to separate the effect of elasticity of intertemporal substitution (EIS) and risk aversion in the stochastic differential utility. In particular, the first term, $f(C, J)$, depends only on EIS, whereas the second term, $\frac{1}{2}\frac{\alpha}{J}\left\|\boldsymbol{\sigma_{J,t}}\right\|^2$ depends only on risk aversion and is linear in it (it might depend on the EIS indirectly through the $\boldsymbol{\sigma_{J,t}}$ term, however).

I further extend the utility specification when the risk-aversion parameter α is stochastic. The specification results in stochastic differential utility being linear in risk aversion and therefore tractable. In particular, it preserves the homogeneity property of the value function and thus allows me to scale everything with the level of total capital.

Finally, agents face a total wealth constraint, which is given by

$$dW_t = \left[W_t\boldsymbol{\theta_t'}\boldsymbol{\lambda_t} + W_t r_t - C_t\right]dt + W_t\boldsymbol{\theta_t'}\boldsymbol{\sigma_R}d\boldsymbol{Z}, \tag{6}$$

where W denotes total wealth, $\boldsymbol{\theta} = (\theta_0,\ \theta_1)^\top$ denotes the vector of shares of wealth in assets that span the market, $\boldsymbol{\lambda}$ denotes a 2×1 vector of risk premia on the two assets, r_t is the equilibrium risk-free rate, and $\boldsymbol{\sigma_R}$ is a 2×2 covariance matrix of returns on the two assets. The formulation in Eqs. (3) and (6) describes a standard portfolio allocation problem of an infinitely lived investor who can freely participate in complete financial markets.

2.1.3. State variables

Because of homogeneity of utility specification in Eq. (3), the use of the unnormalized aggregator, and homogeneity of the capital accumulation processes in Eq. (1), I am able rescale all variables by the total capital or total wealth. I establish this result in section A.3.1. I need therefore only two state variables: the share of capital in the high-risk technology, which I will denote with x, and risk aversion, α.

The risk-aversion process. The process for risk aversion α_t is taken as exogenous,

$$d\alpha_t = \phi(\bar{\alpha} - \alpha_t)dt + \alpha_t\boldsymbol{\sigma_\alpha}d\boldsymbol{Z_t}, \tag{7}$$

where $\boldsymbol{\sigma_\alpha} = (\lambda\varsigma_K,\ \varsigma_\alpha)^\top$ is a vector of loadings on the capital and risk-aversion shocks $\boldsymbol{Z_t} \equiv (Z_K, Z_\alpha)^\top$ in the economy, $\lambda,\ \varsigma_K$, and ς_α are constants, and $\lambda \leqslant 0$ controls the strength of the risk-aversion response to capital shocks. The volatility is scaled by α_t for two reasons. First, it guarantees, α_t is always positive and hence the agents are always risk averse. Second, it makes the size of shocks scale with current level of risk aversion. When α_t is high, shocks to risk aversion matter more than when α_t is low. This assumption is standard in the literature. Campbell and Cochrane (1999) and Bekaert et al. (2010), for example, specify the process for the consumption surplus ratio and risk aversion in logs, which produces heteroskedasticity of the same kind in levels.

This parameter can be interpreted as either time-varying risk aversion (Campbell and Cochrane, 1999; Dew-Becker, 2014) or time-varying ambiguity aversion with respect to model specification (Drechsler, 2013; Hansen and Sargent, 2008).[4]

The share of capital in the high-risk technology. Let the share of capital in the high-risk technology to total capital be $x = \frac{K_1}{K_0 + K_1} = \frac{K_1}{K}$, where $K \equiv K_0 + K_1$ is total capital. With this specification $x \in [0, 1]$ with $x = 0$ corresponding to the case with only low-risk technology and $x = 1$ corresponding to the case with only high-risk technology. Note $x = 0$ and $x = 1$ states are absorbing: once one technology vanishes, it cannot be rebuilt, because loadings on shocks as well as the drift (due to homogeneity of the adjustment-cost function) in Eq. (1) are proportional to the stock of capital, which is zero.

I apply Ito's lemma in Supplementary Materials: Appendix to show that the share of high-risk technology x in equilibrium follows an endogenous stochastic process:

$$dx_t = x_t(1 - x_t)\left[\phi_1(i_{1,t}) - \phi_0(i_{0,t}) - x_t\varsigma_K^2\right]dt + x_t(1 - x_t)\varsigma_K dZ_{K,t}. \tag{8}$$

[4] Variability in α generates time-varying prices of risk in the model. The main results survives if we instead model time-varying quantity of risk (through time-varying volatility of consumption growth).

2.1.4. Competitive equilibrium

Definition 2.3. Given the vector of state variables $\boldsymbol{X} = (x,\ \alpha)^\top$, the SDF process $\frac{d\Lambda}{\Lambda}$ and price functions $\{p_n \equiv p_n(\boldsymbol{X})\}_{n=0}^1$, the total wealth of the representative agent W, the vector of risk premia $\lambda(\boldsymbol{X})$, the risk-free rate $r(\boldsymbol{X})$, and the covariance matrix $\boldsymbol{\sigma_R}(\boldsymbol{X})$ of returns on two assets that span the markets, a *stochastic competitive equilibrium* is a set of allocations $\{C,\ \theta,\ I_0,\ I_1,\ K_0,\ K_1\}_{t=0}^\infty$ such that

(i) agents maximize utility given by the stochastic differential utility process in Eq. (3), subject to their budget wealth evolution in Eq. (6), by choosing how much to consume, C, and how much to invest in either of the two assets, $\boldsymbol{\theta}$;
(ii) firms solve an optimal investment problem in Eq. (2) subject to Eq. (1);
(iii) risk aversion follows an exogenous stochastic process given by Eq. (7);
(iv) capital accumulation grows according to Eq. (1);
(v) aggregate wealth evolves according to Eq. (6);
(vi) financial markets clear, $\boldsymbol{\theta} = \left(\frac{p_n(\boldsymbol{X})K_n}{W}\right)^\top$, $n = \{0,\ 1\}$.

2.1.5. Solution

The first and the second Welfare Theorems hold in the model. I therefore start by solving the planner's problem and then decentralizing the economy to find prices.

The planner chooses investment and consumption to maximize the agent's lifetime utility. Due to homogeneity, I guess that the solution is linear in total capital: $J(K_0, K_1, \alpha) = K \times F(x, \alpha)$. I establish the following result in Supplementary Materials: Appendix, A.3.

Theorem 2.1. *The solution to the planner's problem is given by the system of a PDE,*

$$\frac{\delta}{\rho}\left[\left(\frac{c(x,\alpha)}{F(x,\alpha)}\right)^{\rho} - 1\right] - \frac{1}{2}\alpha\|\boldsymbol{\sigma_F}\|^2 + (1-x)\left(1 - \frac{F_x}{F}x\right)\phi_0(i_0) + x\left(1 + \frac{F_x}{F}(1-x)\right)\phi_1(i_1)$$
$$+ \frac{F_\alpha}{F}\phi(\bar{\alpha} - \alpha) + \frac{1}{2}\frac{F_{xx}}{F}(1-x)^2x^2\varsigma_K^2 + \frac{1}{2}\frac{F_{\alpha\alpha}}{F}\alpha^2\|\boldsymbol{\sigma_\alpha}\|^2 + \left[\frac{F_\alpha}{F} + \frac{F_{x\alpha}}{F}(1-x)\right]x\alpha\lambda\varsigma_K^2 = 0, \quad (9)$$

with boundary conditions listed in Supplementary Materials: Appendix, section A.3.3, first-order conditions for optimal investment,

$$\delta\left(\frac{c}{F}\right)^{\rho-1} = (F - F_x x)\phi_0'(i_0) \quad (10)$$

$$\delta\left(\frac{c}{F}\right)^{\rho-1} = (F + F_x(1-x))\phi_1'(i_1), \quad (11)$$

and the aggregate resource constraint

$$c = (A_0 - i_0)(1-x) + (A_1 - i_1)x, \quad (12)$$

where F_x denotes a derivative of the rescaled value function w.r.t. x, $\boldsymbol{\sigma_F} = \left(1 + \frac{F_x}{F}(1-x)\right)x\boldsymbol{\sigma_K} + \frac{F_\alpha}{F}\alpha\boldsymbol{\sigma_\alpha}$, and $\boldsymbol{\sigma_K} = (\varsigma_K,\ 0)^\top$.

Proof. Supplementary Materials: Appendix, section A.3 provides the details and derivations. □

Next, I decentralize the economy and solve a portfolio allocation problem, which is detailed in Supplementary Materials: Appendix, section A.2.2. The following theorem summarized the result for the SDF of the economy.

Theorem 2.2. *The stochastic discount factor (SDF) of the economy is given by*

$$\frac{d\Lambda}{\Lambda} = -r(\boldsymbol{X})dt + \mathcal{L}(dln f_C - \alpha_t dlnJ)d\boldsymbol{Z}, \quad (13)$$

where $r(\boldsymbol{X})$ is the equilibrium interest rate, $\mathcal{L}(d\tilde{s})$ denotes the vector of loadings on shocks of a stochastic process $d\tilde{s}$, and f_C is the derivative of the flow utility in Eq. (3) with respect to consumption C.

Proof. See Supplementary Materials: Appendix, section A.6 for the derivation. □

2.1.6. Returns

I choose two assets that span the markets as follows. The first asset's instantaneous total return process is given by

$$dR_{0,t} = \frac{A_0 - i_{0,t}}{q_{0,t}}dt + \phi_0(i_{0,t})dt + \frac{dq_{0,t}}{q_{0,t}}, \quad (14)$$

where $q_{0,t}$ is the Tobin's q of low-risk technology. The expression gives the total return on the low-risk technology.

The second asset corresponds to the total return on the high-risk technology. The instantaneous return is given by

$$dR_{1,t} = \frac{A_1 - i_{1,t}}{q_{1,t}} dt + \phi_1(i_{1,t})dt + \frac{dq_{1,t}}{q_{1,t}} + \left(\frac{dK_{1,t}}{K_{1,t}} - \mathbb{E}\frac{dK_{1,t}}{K_{1,t}}\right) + \left\langle \frac{dq_{1,t}}{q_{1,t}}, \frac{dK_{1,t}}{K_{1,t}} \right\rangle. \quad (15)$$

The first component is the dividend-price ratio. The second is a capital increase due to new investment. The third gives the change in the value per unit of installed capital. The fourth is the change in the return due to the shock to the physical capital. The $\langle \cdot \rangle$ notation in the last component denotes Ito's terms stemming from the cross-products of shocks.

The final step to complete the mapping to the competitive equilibrium in Definition 2.3 is to compute the risk-free rate. Supplementary Materials: Appendix, section A.3.1 provides details on these calculations.

2.1.7. Stylized model

Unfortunately, the planner's problem does not have an analytic solution in general. In the Supplementary Materials: Appendix, section A.4 I show pseudo-analytic solutions to the stylized model in which EIS is set to 1 and installation function $\phi_n(\cdot)$ is the same for two technologies and takes a log form. The solution relies on small-noise expansions (perturbations around the non-stochastic steady state). It allows me to establish analytic propositions characterizing the economy which I briefly summarize here:

1. Shocks to risk aversion that are orthogonal to shocks to the capital-accumulation process move returns on high-risk and low-risk technologies in *opposite* directions;
2. Shocks to the risky capital-accumulation process that are orthogonal to shocks to risk aversion move returns on high- and low-risk technologies in the *same* direction;
3. There exists a calibration of the model that produces a positive correlation between returns on low-risk and high-risk technologies for low levels of risk aversion and negative correlation when risk aversion is high.

For more formal argument and proofs, refer to Supplementary Materials: Appendix, section A.4. In Section 3 I will also show evidence that these propositions work numerically in a calibration of the full model that does not rely on the simplifying assumptions above.

2.2. Pricing bonds and stocks

The price of a real long-term bond of maturity T is given by

$$P_{B,t}^{(T)} = \mathbb{E}_t\left[\frac{\Lambda_T}{\Lambda_t} \times 1\right]. \quad (16)$$

In models of such complexity, analytical solutions for the term structure are typically unavailable. I therefore take a different approach that allows me to use pseudo-analytical solutions for returns on the two technologies as good approximations of bond and stock prices.

The method relies on a specific choice of technologies I have made. The returns on the low-risk technology, given by Eq. (14), are similar to the returns on a perpetuity that pays $A_0 dt$ at each instant, which can be defined as $dR_c = \frac{A_0}{q_c}dt + \frac{dq_c}{q_c}$. The difference lies in the investment adjustment, $\left[-\frac{i_0}{q_0} + \phi(i_0)\right]dt$, which reflects the fact that not all of the new investment is installed at the marginal cost. In the formula for the low-risk technology, the "profits" due to this fact are split among all existing shareholders equally. In practice, the adjustment turns out to be small and thus has little impact on the dynamics of returns on the low-risk technology. This allows me to use a solution for the returns on the low-risk technology as a good approximation of returns on a perpetuity that pays $A_0 dt$ every instant. I verify that the dynamics of returns of the low-risk technology are indeed similar to those of a perpetuity, which in turn are similar to those of a long-term real default-free bond, which I price in the model by explicitly computing the expectation of the SDF in Eq. (16). In the calibrated model, which I discuss in Section 3, the correlation between returns on the low-risk technology and a 20-year real bond is above 98%. I am therefore able to use an analytic solution for returns on the low-risk technology as a good approximation of the returns on a long-maturity bond.

I define a stock as a leveraged claim to a portfolio of two technologies. I assume the amount of leverage is time-varying and determined by the relative sizes of the two technologies (the Modigliani-Miller theorem holds in my model; therefore, any amount of leverage is consistent with firms' capital-structure decisions). As a consequence, the aggregate leverage of the economy is pinned down by the aggregate risk-taking and general equilibrium, rather than an individual firm's decision. The asset being shorted in the leveraging process in this case is a perpetuity (a claim on the low-risk technology) rather than an instantaneously risk-free bond. With this definition of the leveraging process, the returns on a stock are equal to returns on the high-risk technology itself. To make this interpretation viable quantitatively, I calibrate the process for capital accumulation in Eq. (1) and other parameters of the model in such a way that the volatility and dynamics of returns and risk premium produced by the high-risk technology closely resemble the respective dynamics generated by the stock market index. With this interpretation, I can therefore use an analytical solution for returns on the high-risk technology as a good approximation to returns on the stock market index.

Note, however, although the specific assumption on technologies made facilitates the analysis and allows me to get approximate analytic formulas for bond and stock prices, the mechanisms of the model do not require one technology to have no exposure to capital shocks. As long as two technologies differ in their exposure to the capital shock, the main results of the paper continue to hold. In this case, I can numerically solve a model with two high-risk technologies that differ in their amounts of cash-flow risk, and price long-term bonds by explicitly computing the expectation of the SDF in Eq. (16). Because the low-risk and high-risk technologies differ in the amounts of their cash-flow risk and returns on the technologies co-move more strongly with returns on bonds and stocks respectively, changes in the relative sizes of the technologies affect the prices of bonds and stocks (through an SDF).

2.3. Mechanisms

With this intuition in mind, I now use the analytical results summarized in Section 2.1.7 and formally derived in Supplementary Materials: Appendix, section A.4, to revisit the main mechanisms of the model.

The technology diversification mechanism relies on having two technologies that differ in their cash-flow risk, in positive supply, and adjustment costs to investment. With an endogenous investment choice, a time-varying endogenous supply of risk emerges. When one technology has a good shock, investors want to rebalance (diversify) to the other technology, but face adjustment costs, which drives up the price of the other technology. Prices adjust because quantities are fixed in the short run. In fact, I show in Section 3 that the model behaves as “two trees” of Cochrane et al. (2008) in the short run, when quantities are essentially fixed, and as Cox et al. (1985) model in the longer run, when quantities are free to adjust and prices stay constant. Unlike “two trees,” however, the mechanism endogenizes the size of technologies and the aggregate risk-taking decision. An endogenous size is important for preserving the stationarity of the model and sustaining an equilibrium in which two technologies persist. An endogenous risk-taking leads to an SDF that produces co-movement of bond and stock returns in the model. It also produces realistic investment dynamics in the model.

The “two trees” mechanism of Cochrane et al. (2008) and Martin (2013) tends to induce a co-movement of returns on any “trees”, whether they are i.i.d. or different in some important dimension. I assume technologies differ in their amounts of cash-flow risk, which leads to a co-movement of returns on low- and high-risk technologies. Because a low-risk technology is more “bond-like” (has lower cash-flow risk), its returns co-move much stronger with returns on a perpetuity (and hence with returns on a long-term bond) than returns of a high-risk technology, which is more “stock-like”. This built-in feature of the model design, delivers an SDF that generates co-movement of bond and stock returns that are priced in the model using the SDF.

With only the technology diversification mechanism present, the model produces positive correlation of bond and stock returns, on average, and positive term premium. Indeed, with symmetric trees and power utility, Martin (2013) shows that excess returns on a perpetuity are always positive and that returns on trees always co-move. The intuition, as he explains is that “bonds are risky because bad times—bad news for the larger asset—are associated with the state variable moving toward $x = 1/2$, and hence with a rise in the riskless rate [which peaks at that point] and a fall in bond prices.” Cochrane et al. (2008) in the log-utility case, also finds that trees co-move, yet demonstrates that in the case when trees are different in terms of their productivity and risk, it is possible to obtain time-varying and at times negative term premium in a two-tree model alone (even without shocks to risk aversion). The intuition in this case is that when a perpetuity is in positive supply and when its share becomes large, interest rate can fall due to lower overall productivity, so that its price increases even as its share goes up (in response to a negative shock to the other asset). Quantitatively, however, this effect appears to be small and difficult to interpret due to highly non-stationary share dynamics in their model in this case.

The flight-to-safety mechanism relies on exogenous variation in risk aversion. When risk aversion rises, investors want to move from the riskier to the less risky technology, which produces a negative correlation between returns on the technologies. The mechanism produces time-varying preference for risk. With only flight-to-safety mechanism present, the model would produce negative correlation of bond and stock returns, and negative term premium. Overall, the general idea behind this mechanism goes back to Constantinides (1990) and Campbell and Cochrane (1999), who studied models with internal and external habits, capable of generating endogenous fluctuations in the curvature of the value function. This is an amplification mechanism, which in models with positively autocorrelated consumption leads a highly negative term premium.[5]

I show that the model-implied consumption growth is positively autocorrelated, consistent with empirical evidence. However, the covariance of contemporaneous consumption growth with the infinite sum of all future expected consumption growth is negative. This property, coupled with Epstein-Zin preferences, tends to generate a positive component to real bond term premia and positive correlation between returns on real bonds and stocks.

The main contribution of this paper is to develop a framework where one of or the other effect can dominate, thus delivering interesting time-correlations of real bonds and stocks, as well as a realistically time-varying term premium in a general equilibrium model. This time-variation and relative strength of the two mechanisms determines the overall correlation between bond and stock returns as well as the sign of the term premium. When risk aversion is low, the technology diversification mechanism dominates, producing a positive correlation between bond and stock returns on average. During

[5] The intuition behind this result can be easily illustrated with a simple consumption-based model with log utility. In such a model a one-period bond price is $\log P_t^{(1)} = -\mathbb{E}_t[\Delta c_{t+1}]$ and term premium on a two-year bond is thus $brx_t^{(2)} = -\text{cov}_t\left(\text{SDF}_{t+1}, \log P_{t+1}^{(1)}\right) = -\text{cov}_t\left(\Delta c_{t+1}, \mathbb{E}_{t+1}[\Delta c_{t+2}]\right)$. When consumption growth is positively auto-correlated (as in the data), bond risk premium is always negative.

those times, bonds are exposed to the same discount-rate risk as stocks, and this exposure is higher than opposing hedging motives. Bonds, therefore, tend to command mildly positive term premium during tranquil times. On the other hand, when risk aversion rises, the flight-to-safety mechanism becomes stronger and often overturns the technology diversification mechanism, to produce a negative correlation between bond and stock returns. During such times, bonds are typically perceived as good hedges against the stock market risk and the hedging motive dominates the common exposure to a discount-rate risk. Bonds thus command a negative risk premium.

The change in the relative importance of two mechanisms occurs because in response to a capital shock, discount rates on the risky asset move in a way that dampens the cash-flow effect on the asset. This dampening becomes stronger as risk aversion rises, leading to a relatively weaker comovement of bond and stock returns. At the same time, an increase in risk aversion leads to a stronger "decoupling" of bond and stock returns due to flight-to-safety. As a result, the flight-to-quality mechanism starts dominating the technology diversification mechanism at higher levels of risk aversion.

3. Dynamics

I now explore the dynamics the general model generates, and verify the results obtained are in line with the intuition that we acquired by analyzing small-noise expansions of the stylized model.

3.1. Solution method

The model is solved numerically by finding a solution to the system of equations in Theorem 2.1 using high-order projection methods. I parameterize the value function and two investment functions as a complete product of 20th-order Chebyshev polynomials in two state variables, x and α. Once the approximation to the value and investment functions are found, I use them to calculate the aggregates and prices in the model. Supplementary Materials: Appendix section A.7.1 provides further details.

3.2. Calibration

Parameter values are assigned based on three types of criteria. First, a set of parameters is chosen to match standard values used in the literature. This is done to make the paper most comparable to other related research, and to make sure the results are not driven by exotic parameter choices. Second, some parameters are intentionally restricted to shut down specific second-order mechanisms in the model in order to simplify the exposition and facilitate the analysis of the key economic forces in the model; this choice has an additional benefit in making sure the model is not overly parameterized. Lastly, all remaining parameters are estimated minimizing a global objective to yield the best implications for key asset pricing moments. Because the majority of parameter choices fall into the last category, the overall approach to calibrating the model resembles estimation.

3.2.1. Details of calibration

Time-discounting $\delta = 0.03$ and EIS $\psi = 2$ parameters are pre-specified to match standard values used in the literature (see Koijen et al., 2017, Van Binsbergen et al., 2012). The parameter which governs propagation of capital shocks is restricted to zero, $\lambda = 0$. This is done to have a clear separation between shocks to capital and risk aversion, and to learn the implications of these shocks for the two technologies. Furthermore, parameters θ and ξ in the installation cost function $\phi_n(i_n) = \xi \times \ln\left(1 + \frac{i_n}{\theta}\right)$ of both technologies are restricted to be the same, in order to make sure that equilibrium differences between the two technologies are only due to their differences in marginal product of capital (MPK) and exposure to shocks. The rest of the parameters are estimated as to minimize the distance from moment targets. I discuss these targets in the next section.

Due to the highly non-linear nature of the model and a relatively large number of parameters to be estimated (eight) the estimation process is non-trivial. I now provide a short summary of this process.

The estimation is conducted in six steps. First, I draw each of the eight parameters independently and randomly from the normal distribution. Parameters of these distributions were chosen through an iterative process which started with a relatively wide range for each parameter and narrowed down the values as to produce solutions and moments in admissible ranges. Second, for each of the many sets of parameters values, the model is solved and the implied model's quantities are recorded. Third, state processes of each of the model's solutions are simulated for 100,000 periods and the models' moments are computed. Models with moments outside admissible ranges are discarded. Fourth, I estimate a neural network, as a device for non-linear function approximation, which allows me to create a high-dimensional mapping function from the set of plausible parameter values to the set of model-implied moments. Parameters of the network are optimized to fit the objective function (distance between the network-predicted and model-implied moments) in the least squared sense. Fifth, I differentiate the network with respect to inputs (model's parameter values) and optimize over the model's parameter values to minimize the distance between predicted moments and empirical targets. In practice, because neural networks are randomly initialized, the last two steps are repeated 100 times to estimate an ensemble of neural networks with their predictions subsequently averaged out. Lastly, the model is re-estimated for the choice of parameter values which maximize the objective and its moments are recorded.

Supplementary Materials: Appendix section A.7.6 provides further details on how the neural network is estimated.

Table 1
Calibration. Annualized parameters used in the calibration of the general model from Section 2.1.

	Variable	Value
	Preferences	
Time discouting	δ	0.03
EIS	ψ	2
Mean reversion of α	ϕ	0.29
Mean of risk aversion	$\bar{\alpha}$	25
Volatility of risk aversion	ς_α	0.14
Propagation of capital shocks	λ	0
	Technology	
Volatility of capital	ς_K	0.065
MPK of bond technology	A_0	0.029
MPK of stock technology	A_1	0.079
Adjustment cost of riskless technology	θ	0.024
Adjustment cost of risky technology	ξ	0.05

Table 2
Fitted moments. The Data column shows empirical estimates of moments. The Model column shows the values implied by the calibration of the model in Table 1. All moments are annualized.

	Data	Model
Mean consumption growth	2%	2%
Standard deviation of consumption growth	2.1%	3.5%
Mean output growth	1.9%	2.1%
Standard deviation of output growth	2.6%	4.3%
Mean price-dividend ratio	32	29
Standard deviation of the log price-dividend ratio	0.43	0.4
Mean equity risk premium	6.6%	4.7%
Standard deviation of equity returns	16%	7.9%
Mean equity Sharpe ratio	0.42	0.57
Mean monthly real risk-free rate (annualized)	1.0%	2.4%
Standard deviation of real risk-free rate	0.7%	0.81%
Mean total wealth excess return	2.2%	2.8%
Mean wealth-consumption ratio	87	67

3.2.2. Moments

Table 2 reports empirical and fitted values of the moments used in calibration. Estimated values for mean return on total wealth and mean wealth-consumption ratio are taken from Lustig et al. (2013). The rest of the moments are estimated using data from FRED and CRSP.

Overall, the model is able to fit the moments reported in Table 2 both qualitatively and quantitatively, with the exception of high volatilities of returns on stocks, which could be fixed by applying additional leverage to claims to the high-risk technology.

3.3. Results

3.3.1. Conditional distributions of major moments

To construct conditional distributions, I perform Monte-Carlo simulations starting from a steady state (unconditional means of state variables) and show histograms of variables of interest. The histograms give a clear understanding of where the economy “lives” in the state space and what is the distribution of variables of interests. Additionally, because they are conditional histograms, we can gain insights about how the level of risk aversion affects real and financial variables in the model.

Fig. 2 depicts conditional distribution of state variables, interest rate, and return correlation of the economy. Disregarding the colors, each plot shows a histogram of a respective variable. Different colors within each bar of a histogram show what fraction of observations that contributed to the bar had risk aversion within a certain percentile. For example, yellow (bright) depicts observations with risk aversion being in the top 20 percentile, whereas red (dark) depicts observations with low risk aversion in the bottom 20 percentile. Because the sort is made on risk aversion, the first histogram that shows the distribution of the risk-aversion parameter naturally orders the colors according to their level of risk aversion, from red (dark) on the left to yellow (bright) on the right. The second plot shows the distribution of a state variable x. We can

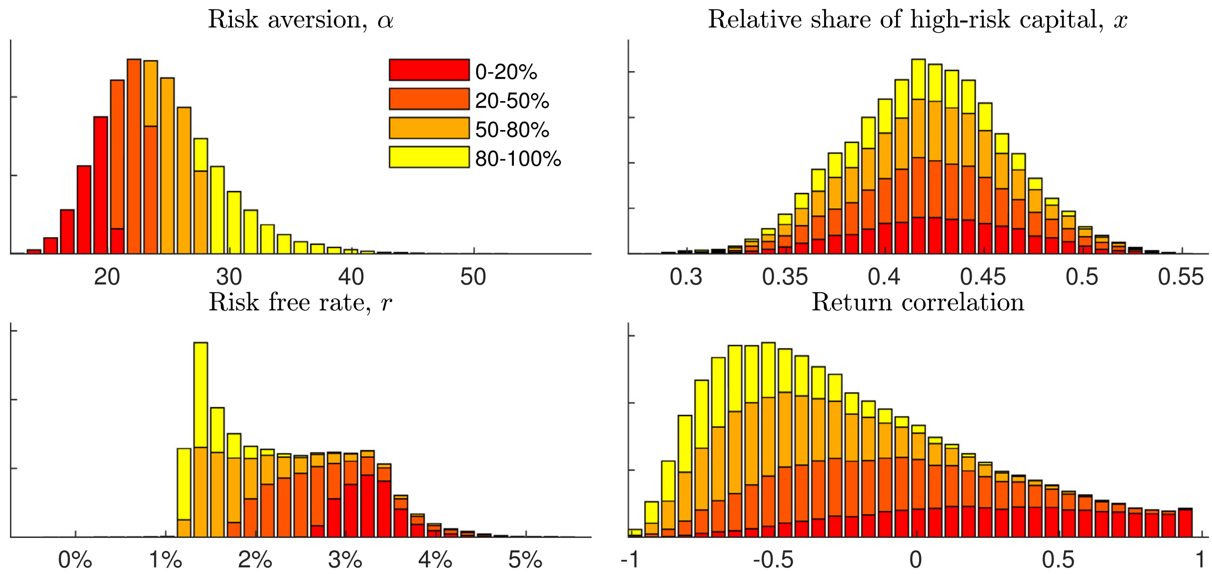

Fig. 2. Conditional distributions of state variables, interest rate, and Bond-Stock correlations. Each color within a bar shows a fraction of simulated observations with risk aversion within a given percentile.

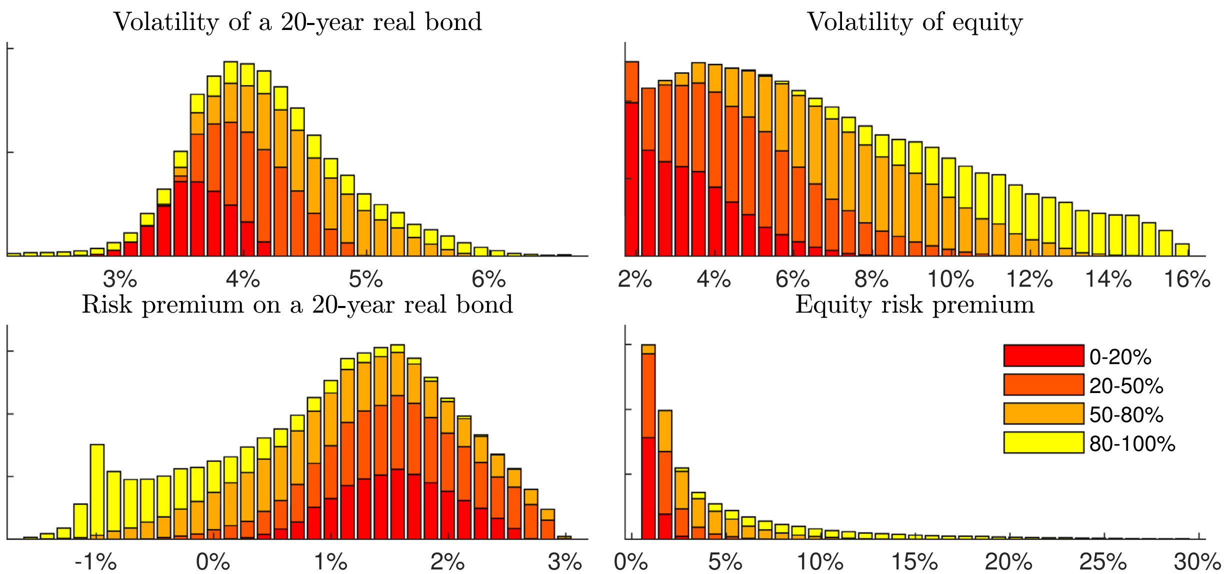

Fig. 3. Conditional distributions of financial variables. Each color within a bar shows a fraction of simulated observations with risk aversion within a given percentile.

see the high-risk technology has a mean of about 43% of total capital.[6] Unlike Cochrane et al. (2008), the model therefore produces a stationary distribution of capital with both technologies coexisting in equilibrium. The bottom-left figure shows real interest rate varies from around 0% to 6% annually. It is high when risk aversion is low, and low when risk aversion is high, consistent with the empirical evidence. The bottom-right histogram depicts the distribution of correlations between returns on high-risk and low-risk technologies. It is highly time-varying, changing signs, and is more likely to be high and positive when risk aversion is low, and low and negative when risk aversion is high. Supplementary Materials: Appendix section A.7.7 shows the same distributions conditional on the share of high-risk capital, x.

Fig. 3 shows distributions of bond and stock volatility and risk premia. Stock returns are more volatile due to cash-flow shocks. Both bond and stock returns are more volatile when risk aversion is high. Sharpe ratios (not shown), however, go

[6] Lustig et al. (2013) find the return on total wealth behaves much more like a long-term bond rather than the stock market. This finding is consistent with a relatively lower share of high-risk capital in the total wealth portfolio.

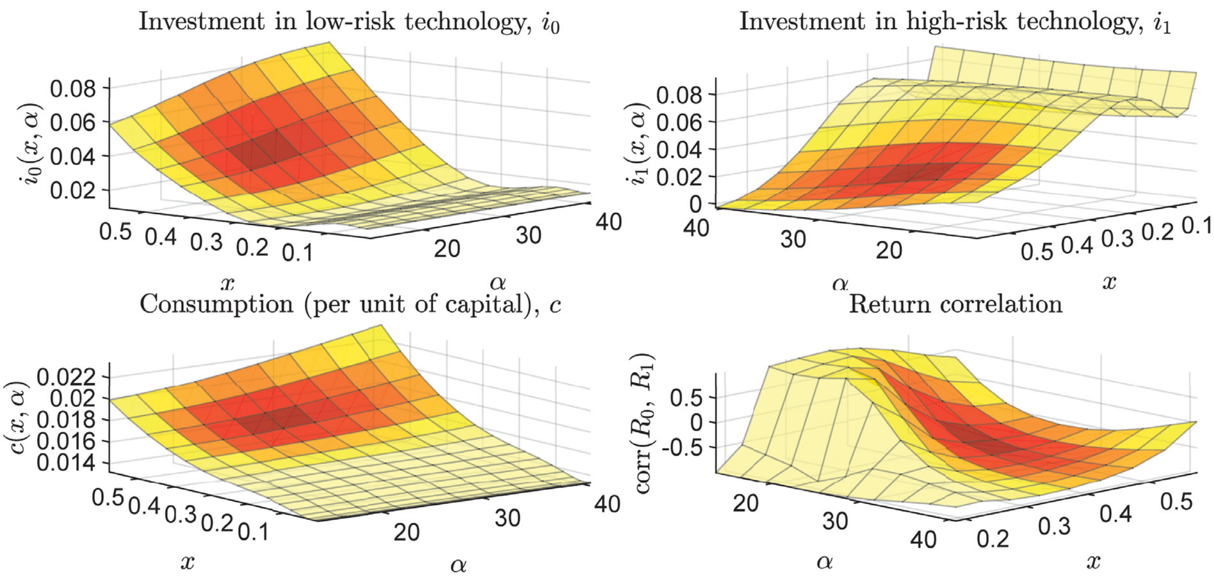

Fig. 4. Policy and transition functions as functions of two state variables, x **and** α**.** Warmer colors depict regions of the state space with higher probability density.

in opposite directions: the Sharpe ratio on stocks is high when risk aversion is high, and is low and negative on bonds. The term premium on a 20-year bond varies from around −2% to about +3% annually. It is negative when risk aversion is high and positive otherwise. Equity risk premium varies from 1% to about 30% annually. It is high when risk aversion is high and low otherwise.

3.3.2. Policy functions

I now discuss policy and transition functions as functions of two state variables, x and α. Policy functions for consumption and investment in the two technologies are shown in Fig. 4. Warmer colors depict regions of the state space with higher probability density.

Investment in the low-risk technology tends to increase when the share of high-risk capital x becomes large, as the economy tries to sustain the stationary distribution of x. Similarly, investment in the high-risk technology increases as x falls. These two forces generate a strong drift toward the mean value of x (conditional on α) and are key for the existence of equilibrium in which two technologies survive. Cochrane et al. (2008) lack such a force in modeling two technologies as trees and thus allowing for no endogenous investment. As a result, their economy is not stationary and one tree always dominates the other. Stationarity in my model is achieved precisely through endogenous capital reallocation (and high risk aversion, as opposed to log utility, which makes mean-reverting forces even stronger).

Investment in the low-risk technology increases in risk aversion α while investment in the high-risk technology falls in α. This pattern of investment is a manifestation of the flight-to-safety mechanism. When risk aversion is high, people tend to reallocate from high- to low-risk assets, driving up the investment and prices on low-risk technology.

Consumption is increasing in the share of high-risk technology x because of higher aggregate productivity when the more productive technology is larger. Consumption is also increasing in risk aversion α, rising slightly as risk aversion increases. With the EIS greater than one, agents smooth out their consumption in time and consume relatively more per unit of capital when risk aversion increases, contributing to a less volatile consumption path. This fact and the aggregate resource constraint imply the aggregate investment per unit of capital must fall as risk aversion rises. When the EIS is fixed at 1 (stylized model), consumption is completely flat in risk aversion.

Finally, the correlation between returns on high-risk and low-risk technology initially falls steeply as risk aversion increases and flattens out later. It is highly positive for low risk aversion and becomes negative for higher values of risk aversion, consistent with Proposition A.3 in Supplementary Materials: Appendix. Correlation also decreases in x when α is low and increases in x for high levels of α. When risk aversion α is low, the flight-to-safety mechanism is weak, especially when we have relatively little high-risk capital. When risk aversion becomes high, however, flight-to-safety becomes the dominant mechanism. As a result, when x is high, the fraction of low-risk technology is low and thus the price impact on the technology in response to capital shocks is magnified, causing the comovement of returns on two technologies increase, and thus making the technology diversification mechanism stronger. Higher x also results in a smaller discount-rate effect on the technology and weaker dampening of the cash-flow effect (will be discussed later), further increasing the comovement of the two technologies.

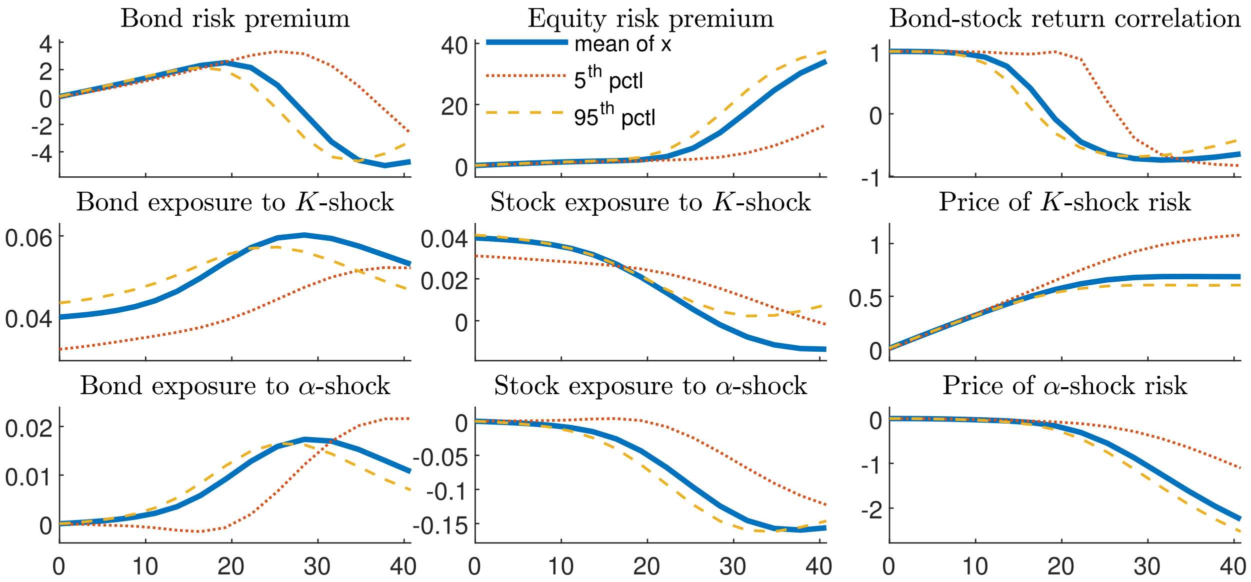

Fig. 5. Impact of risk aversion. Excess returns, loadings of bonds and stocks on two shocks, and prices of risk as functions of risk aversion α for fixed values of x. Three levels of x are depicted with different lines: unconditional mean of x (solid), 5th percentile (dotted), and 95th percentile (dashed). The x-axis shows the level of risk aversion, α.

Fig. 5 shows the excess returns, loadings of bonds and stocks on two shocks, and prices of risk as functions of risk aversion α for fixed values of x. Three different levels of x are considered, each corresponding to a different line: the solid line corresponds to the unconditional mean value of x, the dotted line corresponds to 5^{th} percentile, and the dashed line corresponds to the 95^{th} percentile. The horizontal axis shows the level of risk aversion, α, ranging from 0 to 40 in each plot. The term premium displays a non-monotonic pattern, initially increasing and later falling in α. Equity risk premium monotonically increases in risk aversion for all values of x, consistent with Proposition A.4 in Supplementary Materials: Appendix. Correlation falls in α and is generally high and positive for high levels of x.

Prices of capital risk and risk-aversion risk are monotone in risk aversion. The price of capital risk is increasing and always positive, whereas the price of risk-aversion risk is decreasing and negative. Whereas the loadings of bonds and stocks on a risk-aversion shock mostly increase with risk aversion in absolute value and are of different signs, the loading of stocks on a capital shock falls in the level of risk aversion, because, in response to a capital shock, discount rates on the risky asset move in a way that dampens the cash-flow effect on the asset. This dampening becomes stronger as risk aversion rises, leading to a relatively weaker comovement of bond and stock returns. The weaker comovement contributes to the technology-diversification effect becoming relatively weaker than flight-to-safety, or even reversed, for high levels of risk aversion. As a result, the correlation between bond and stock returns becomes more negative as risk aversion rises.

3.3.3. Impulse response functions

I now analyze the dynamics of the model and response to shocks over time. For each variable of interest, I construct an impulse response by hitting an economy with a contemporaneous shock (1 s.d. in magnitude) and performing Monte-Carlo simulations of the economy 20 years forward to trace the impact of the shock. I then average all Monte-Carlo trajectories to compute the expected conditional response to a shock at the unconditional mean values of state variables conditional on a shock hitting at $t = 0$. I perform similar calculations for the economy with no shock at $t = 0$ to compute the trajectory of the economy that was not hit by the shock. The relative difference of two defines a non-linear impulse response. Formally, for each variable of interest V, I compute an impulse response from $t = 0$ to T as

$$IR_{t\to T}(\boldsymbol{X_t}) = \frac{\mathbb{E}_t[V_T|\boldsymbol{X_t},\ Z_{n,t} = 1] - \mathbb{E}_t[V_T|\boldsymbol{X_t},\ Z_{n,t} = 0]}{\mathbb{E}_t[V_T|\boldsymbol{X_t},\ Z_{n,t} = 0]}, \tag{17}$$

where $\boldsymbol{X_t}$ is a vector of state variables and $Z_{n,t}$ denotes a realization of a shock at $t = 0$.

Because the model is non-linear, these impulse responses cannot be calculated by zeroing out future shocks, because in non-linear models, future shocks interact with future values of state variables and these effects are important for studying the dynamics.

Fig. 6 shows the responses of state variables and key quantities and prices of the model in response to two shocks. A solid blue line shows a response to the capital shock. The dashed red line shows an impulse response to a pure risk-aversion shock. I shock the economy at its mean values of state variables and track responses forward for 20 years.[7]

[7] I also plotted impulse responses at many other points of the state space. Qualitatively, all of them look similar to Fig. 6.

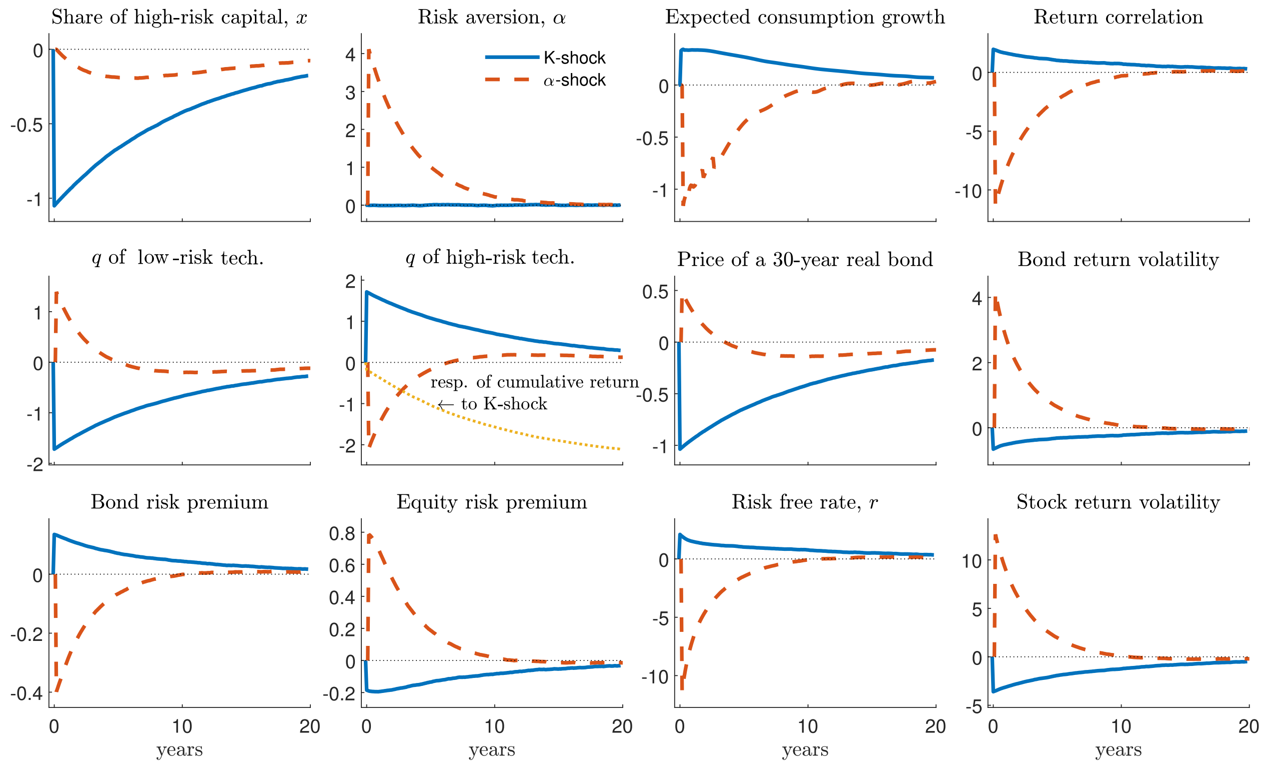

Fig. 6. Impulse responses to a 1 s.d. shock at a stochastic steady state, in %., over 20 years. Responses to two shocks are depicted: a negative capital shock (solid line), and a positive risk-aversion shock (dashed red). Bond and equity excess returns are shown in absolute deviations from their unconditional values. The additional dotted line on the marginal q of high-risk technology plot shows cumulative returns on the technology after the shock.

Let's first focus on the risk aversion shock – the dashed red line. A positive risk aversion shock has no contemporaneous effect on share of high-risk capital x, with small negative impact in the following 20 years. The shock, however, has a strong effect on instantaneous returns of two technologies: marginal q of low-risk technology rises on impact, producing positive realized returns while marginal q of high-risk technology falls, delivering negative realized returns. Instantaneous risk premia on two technologies go in the opposite direction: they fall for low-risk technology and rise for high-risk technology (two bottom-left plots). This effect is a manifestation of the flight-to-safety phenomena. Finally, both volatilities increase on impact, and the risk-free rate and correlation of returns falls.

The blue solid line depicts the response to a negative capital shock. Risk aversion does not respond on impact. Marginal q of the low-risk technology falls, whereas that of the high-risk technology rises. The loss of capital (cash-flow effect) negatively affects contemporaneous returns on the high-risk technology, however. To illustrate this effect, I plot an additional line in the figure for marginal q of high-risk technology that shows the cumulative return on the technology. It substantially diminishes or even reverses the effect of q going up, on impact, as a result of the loss of capital. The effect gets stronger at longer horizons, since the cumulative return continues to decline as time passes. Returns on both technologies therefore co-move in response to a capital shock – the technology diversification mechanism. The figure also shows that in response to capital shock, discount rates on the risky asset move in a way that dampens the cash-flow effect on the asset (marginal q of high-risk technology goes up, whereas the total return falls). This effect makes the flight-to-quality mechanism dominate the technology diversification mechanism at higher levels of risk aversion and therefore achieving the changing sign of correlation of bond and stock returns.

Unlike the risk-aversion shock, capital shock causes the risk-free rate to rise, both volatilities to fall, and correlation to rise. Expected consumption growth stays positive for more than 20 years. This persistently positive expected consumption growth (compared to the fast-reverting-to-zero expected consumption growth in response to the risk-aversion shock) leads to a negative covariance of contemporaneous consumption growth and the sum of all future expected consumption growth. Coupled with Epstein-Zin preferences, which make agents care about long-run future consumption growth, it tends to generate positive real bond risk premia at long horizons. At short horizons, however, autocorrelation of consumption growth in positive, consistent with empirical evidence.

I verify that impulse responses of a 30-year zero-net-supply default-free real bond that I price in the model using an SDF, are similar to impulse responses of the low-risk technology's price (Tobin's q). Two plots in the middle row (first and third) show both respond similarly to either of the shocks. Similar responses to two shocks imply that the dynamics of returns on

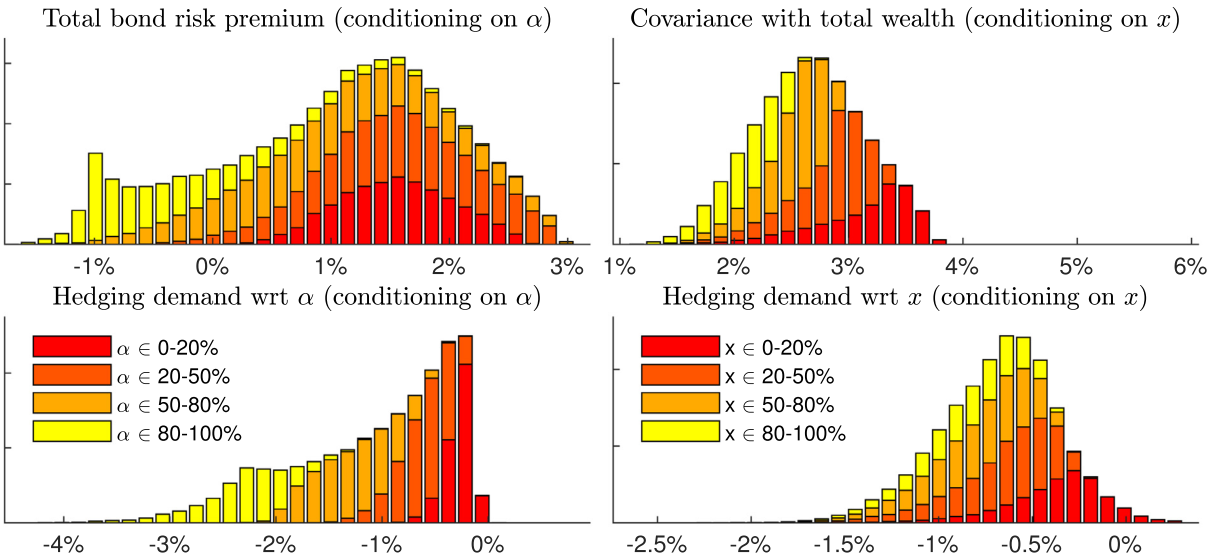

Fig. 7. ICAPM decomposition of bond risk premium. Left-top figure shows a conditional histogram of the total term premium on a 20-year real bond. The other three figures show conditional histograms of three components of the risk premium: a component due to covariance with the total wealth portfolio (CAPM term) and two ICAPM hedging demands. Two plots on the left (right) are conditioned on α (x).

two assets should be similar. I also verified the similarity of responses at different points of the state space. Additionally, I find that the correlation between returns on the low-risk technology and a 20-year real bond (which I price numerically) in the simulated data is 98%. These facts confirm the conjecture that I can use an analytic solution for returns on the low-risk technology as a good approximation for returns on actual long-term bonds, which can be priced only numerically.

Impulse responses of risk premia in the bottom row show another interesting property of the model. We can see that in response to shocks, instantaneous risk premia react significantly on impact as the quantity of capital is fixed in the short run and prices adjust. Responses of expected instantaneous risk premia at longer horizons, however, converge to zero, because the capital is flexible in the long run and quantities adjust. This pattern suggests that the model acts like the “two trees” model of Cochrane et al. (2008) in the short run but behaves as a CIR-type model (Cox et al., 1985) in the long run due to its stationarity.

Finally, impulse responses of x and α show two variables have different persistence. Share of high-risk capital x is highly persistent, with a half life of around 10 years. This variable drives endogenous low-frequency variation in the model. Risk aversion α, on the other hand, is much less persistent and drives mostly higher-frequency variation.

Responses to shocks away from the stochastic steady state are qualitatively similar. to those in Fig. 6.

3.3.4. Risk premia decomposition

Theorem 3.1. *The model has the following two-state variable ICAPM representation:*

$$\boldsymbol{\mu_R} - r = \alpha \times cov(dR, dR_{TW}) + \lambda_\alpha \times cov(dR, d\alpha) + \lambda_x \times cov(dR, dx), \tag{18}$$

where $\lambda_\alpha = (\alpha - 1)\frac{J_\alpha}{J}$, $\lambda_x = (\alpha - 1)\frac{J_x}{J}$, $\boldsymbol{\mu_R}$ *is a vector of conditional expected returns on two assets, and* R_{TW} *is the return on the total wealth portfolio.*

Proof. Refer to Supplementary Materials: Appendix section A.6. □

Theorem 3.1 can be used to decompose the conditional bond risk premium into the three components. Fig. 7 performs such a decomposition for a 20-year real bond. The left-top figure shows a conditional histogram of the total term premium on the bond. The other three figures show conditional histograms of three components of the risk premium: a component due to covariance with the total wealth portfolio (CAPM term) and two ICAPM hedging demands. Two plots on the left are conditional on the level of risk aversion α (each color within a bar shows the fraction of observation that had a simulated value of risk aversion within some percentile). Two plots on the right are conditional on the share of capital in high-risk technology x (each color within a bar shows the fraction of observation that had a simulated value of x within some percentile).

The covariance with total wealth portfolio is mostly positive and generates most of the positive risk premium. This component is highly correlated with the share of high-risk capital x (as can be seen from its conditional distribution), whereas the correlation with risk aversion α is low. Most of the technology-diversification effect therefore manifests in the CAPM component. The hedging demand with respect to risk aversion, on the other hand, generates most of the negative

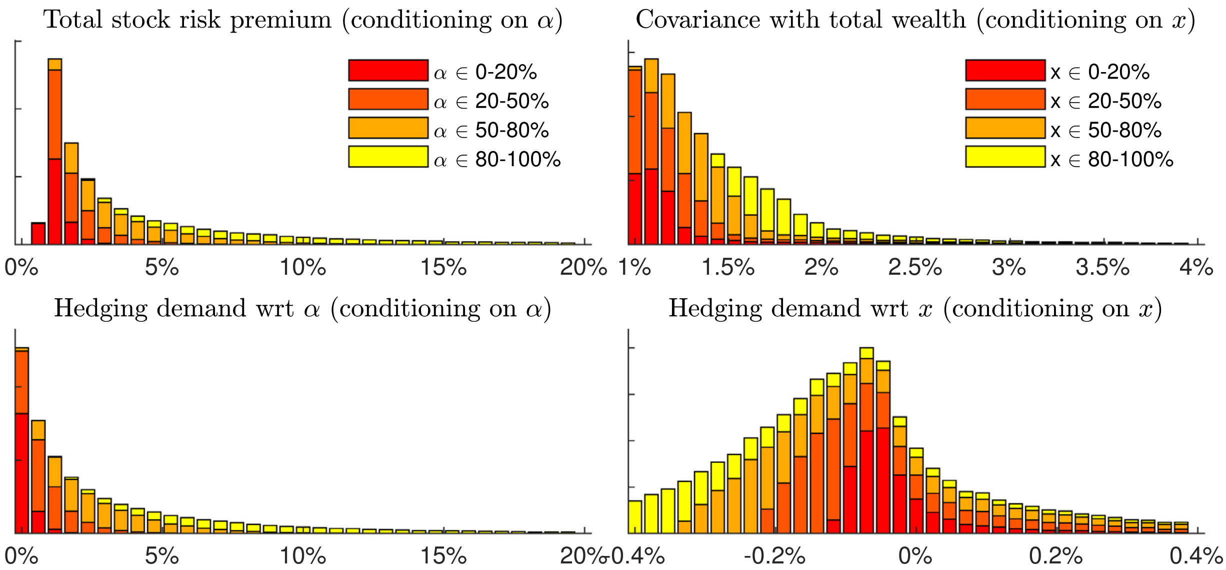

Fig. 8. ICAPM decomposition of stock risk premium. Left-top figure shows a conditional histogram of the total equity risk premium. The other three figures show conditional histograms of three components of the risk premium: a component due to covariance with the total wealth portfolio (CAPM term) and two ICAPM hedging demands. Two plots on the left (right) are conditioned on α (x).

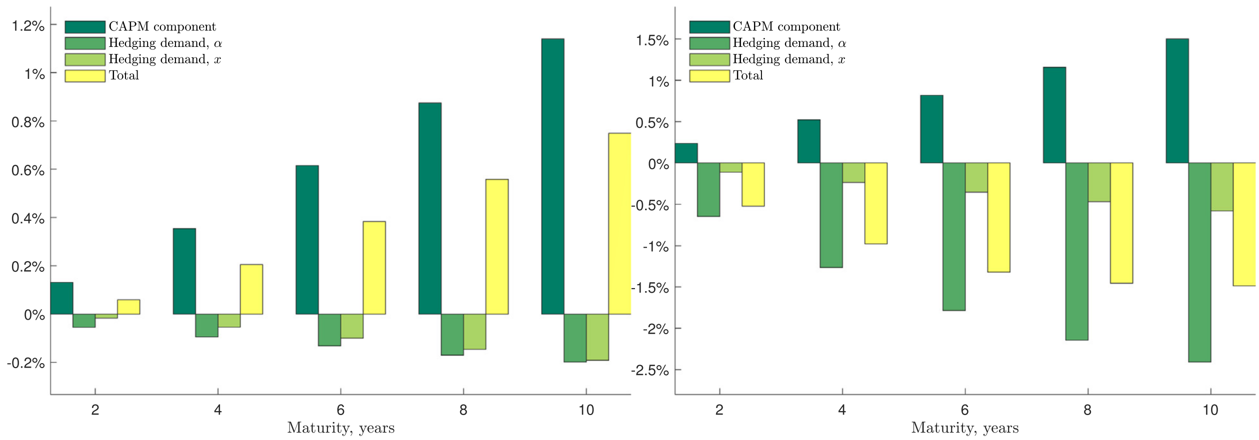

Fig. 9. ICAPM decomposition of average bond risk premium by maturity conditional on the level of risk aversion. Risk aversion is in the bottom 20th percentile (left panel) or top 20th percentile (right panel). The bars show contributions of each of the three ICAPM components to the total bond risk premium (shades of green, labeled) and total bond risk premia (yellow) for each maturity. For each maturity, bar colors are arranged in the same order as legends.

term premium and is highly correlated with risk aversion, which is a manifestation of the flight-to-safety mechanism. The hedging demand with respect to state variable x also contributes to the negative term premium but to a lesser degree.

Fig. 8 shows the same decomposition for stock risk premia. It is primarily driven by the first two components, whereas the contribution of the hedging demand with respect to x is small. Hedging demand with respect to α produces most of the variation of stock risk premia and is the single most important component in determining equity risk premium. Kozak and Santosh (2020) show an empirical factor that proxies for this component captures most of the cross-sectional variation in stock returns.

3.3.5. Term structure of interest rates

Bond risk premia decomposition by maturity

Fig. 9 shows conditional decompositions of average excess returns on real default-free zero-coupon bonds for maturities from 2 to 20 years. Left and right panels of Fig. 9 depict decompositions of excess returns conditional on low risk aversion (bottom 20th percentile) and high risk aversion (top 20th percentile), respectively. Each bond is priced by explicitly calculating the expectation of the SDF at any given maturity.

The plot shows that the total excess return on a bond (yellow bars) increases with maturity and that its two major components are the covariance with total wealth, which is primarily driven by the technology diversification mechanism, and the hedging demand with respect to risk aversion, which is driven by the flight-to-safety mechanism. Two mechanisms require compensation of different signs. Covariance with the total wealth is a positively priced risk (as predicted by the CAPM), and the hedging demand is a hedge (because bonds tend to increase in price in bad times) and thus is negatively priced.

When risk aversion is high (right panel), the flight-to-safety mechanism is strong and dominates technology diversification at all maturities, which results in negative conditional bond risk premia. Similarly, when risk aversion is low (left panel), the technology diversification mechanism dominates, and conditional bond risk premia tend to be high.

Kozak and Santosh (2020) estimate a related three-factor ICAPM with news to discount rates as a factor and find a similar decomposition of term premia by maturity in the data. In particular, they show that bonds earn a large premium for loading on the total wealth risk (the technology-diversification mechanism), whereas they command a large negative premium for loading on their discount-rate factor (the flight-to-safety mechanism). Their ICAPM is able to jointly price cross-sections of bonds and equity portfolios, consistent with the mechanisms of the model in this paper.

Yield curves and expected return curves

The model is able to generate a wide array of yield curve and expected return shapes (Figure B.2 and Figure B.3 in Supplementary Materials: Appendix) observed in the data. The figures show that: (i) yield curves are downward-sloping when risk aversion is low (at maturities up to 15 years) or when x is large (at short maturities), (ii) as risk aversion increases, yield curves turn to slope upwards, (iii) the term structure of expected returns are generally upward-sloping for low-to-medium levels of risk α and x, suggesting that longer-maturity bonds deliver higher risk premia during such times, and (iv) when risk α or x become high, real bonds with maturities up to 10 years can exhibit a negative and downward-sloping term structure; risk premia on the very long-term bonds (20–30 years) is positive though.

4. Empirical evidence

In this section I provide suggestive empirical evidence for the mechanisms of the model and test predictions of the model in the data.

4.1. Data and samples

The data in this section come form multiple sources. Equity returns (daily) and accounting data are from CRSP and Compustat. One of the empirical estimates of the share of high-risk capital x, uses aggregate Current-Cost Net Stocks of Fixed Assets from the Bureau of Economic Analysis (BEA). Treasury and TIPS data come from Gürkaynak et al. (2007) and Gürkaynak et al. (2010), respectively. GZ credit spread is from Gilchrist and Zakrajšek (2012). VIX data are from Cboe. Fama and French industry classification comes from Ken French's website.

Samples start based on availability of the data series from the above sources. GSW nominal yields are available from 1971. Real yields start in 1999. GZ spread is available since 1973. VIX starts in 1990. All samples end in 2020.

4.2. State variable dynamics

4.2.1. Risk aversion

The model predicts that risk aversion is the main driver of variation in financial variables. In particular, we saw in Figs. 2 and 3 that when risk aversion is high in the model, correlation between bond and stock returns is low and stock volatility is high. It is informative to look at observable financial variables that proxy for unobservable risk aversion and see whether they behave in a way consistent with the model's predictions. I focus on three such variables: VIX, realized 3-month volatility, and corporate bond credit spreads.

In Figure B.1 in Supplementary Materials: Appendix I show that, in line with model's predictions, bond-stock correlation (using either real and nominal bonds) is strongly negatively correlated with the three proxies for risk aversion.

I explore statistical significance of these relationships in Table 3, by estimating time variation in bond's beta using empirical financial proxies for risk aversion,

$$R_{B,t} = \alpha + \beta R_{M,t} + \delta (R_{M,t} \times \mathrm{RA}_t) + \varepsilon_{t+1}, \tag{19}$$

where R_B and R_M are daily excess returns on a 10-year nominal treasury and the stock market index, respectively, and RA_t is one of the three proxies for risk aversion described above. I test the null-hypothesis $H_0 : \delta = 0$ and find that $\hat{\delta}$ is negative and statistically significant in each of the regressions. Therefore, times when VIX, Volatility, or Credit Spread are high are also times when correlation between bond and stock returns is relatively low.

4.2.2. Share of high-risk capital

The share of capital in the high-risk technology x drives the low-frequency variation in the model. Using two technologies is a theoretical device to model a continuum of capital of various degree of riskiness across all firms in the economy. The technologies are designed to model cross-sectional heterogeneity in cash-flow risk across firms (e.g., low-risk "utility"

Table 3
Conditional market beta of a bond. The table shows estimates of the regressionwhere $R_{B,t}$ and $R_{M,t}$ are daily excess returns on a 10-year nominal treasury and the stock market index, respectively, and RA_t is one of the three proxies for risk aversion: VIX, volatility, or GZ credit spreads. *t*-statistics in parantheses use HAC standard errors with a small-sample correction.

	$\hat{\beta}$	$\hat{\delta}$	Obs.	R^2
VIX (3-month MA)	−0.29	−0.13	7658	0.05
	(−3.67)	(−2.23)		
3-month Vol.	0.13	−8.90	12,242	0.02
	(2.75)	(−4.01)		
GZ Credit Spread	0.16	−0.07	11,941	0.03
	(3.82)	(−3.94)		

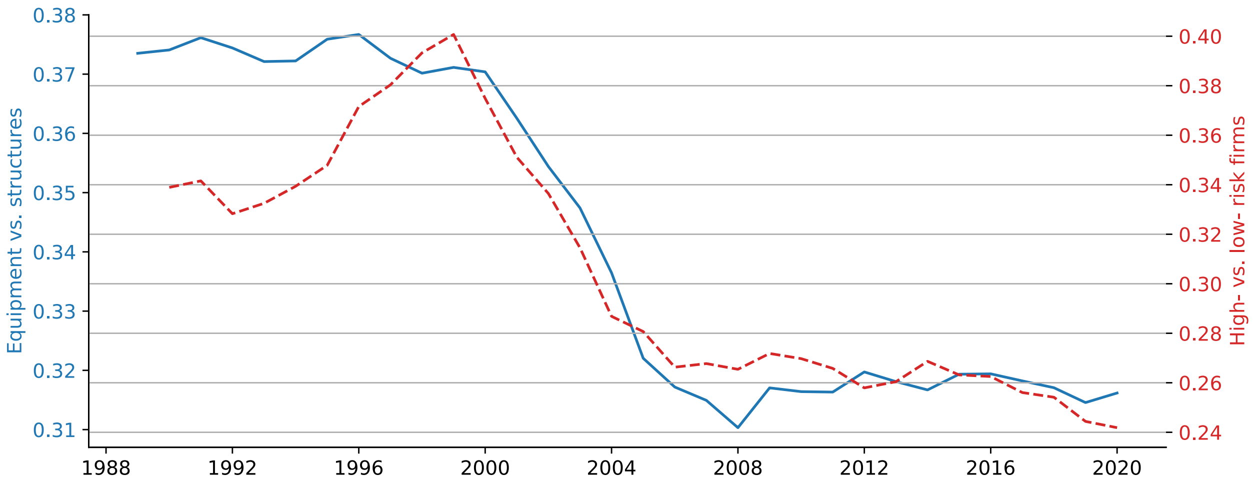

Fig. 10. Estimates of share of high-risk capital *x*. Solid-blue line (left axis) shows estimates of *x* based on the Current-Cost Net Stocks of Fixed Assets from the BEA of equipment (high risk) vs. structures (low risk). Dashed-red line (right axis) estimates *x* using total capital of low-risk (non-durables, utilities, and shops) and high-risk (durables and manufacturing) industries using Fama and French 12-industry classification. Total capital is the sum of net PPE (property, plant, and equipment), non-tangible assets net of amortization, and goodwill.

companies vs. high-risk "hi-tech" companies) or within firms. Therefore, there are two ways to think about an empirical counterpart to such a split: *across* and *within* firms.

I therefore use two different measures of *x*. The first measure is based on aggregate Current-Cost Net Stocks of Fixed Assets from the Bureau of Economic Analysis (BEA) as in Jermann (2010) and Jermann (2013). BEA provides quantity indexes of investment for equipment and software as well as for structures. I use the former as an empirical counterpart for the stock of capital in the high-risk technology, and the latter for the stock of capital in the low-risk technology. This measure has an advantage that it measures the total stock of capital in the economy. In reality firms employ capital of riskiness on a wide spectrum. BEA data allow me to artificially split the capital into two broadly defined categories *within* each firm.

The second measure is based on computing *x across* firms by assuming that some companies employ only high-risk capital, while others use only low-risk capital. I perform this split by assigning each firm from CRSP and Compustat into one of the 12 Fama-French industries. I define low-risk industries as consumer non-durables, utilities, and shops (wholesale, retail, and some services), and high-risk industries as durables and manufacturing. I then aggregate total capital within each group. Total capital is defined as the sum of net PPE (property, plant, and equipment), non-tangible assets net of amortization, and goodwill. With these definitions of low- and high-risk capital, I construct an empirical estimate of *x* as total capital in risky companies divided by the sum of two types of capital. This measure has a disadvantage in that it cannot serve as an estimate of the level of *x* because of an arbitrary nature of the split of quantity of capital into low and high risk, but it can give us a sense of how *x* might be evolving in the economy.

The corresponding dynamics for the BEA measure (solid blue) and the Industry measure (dashed red) are shown in Fig. 10. Recall that the mechanism behind this variable, technology diversification, was responsible for comovement of returns of bonds and stocks and positive real term premium. We observe that the share of high-risk capital *x* was high before 2000s – the period when the correlation between bond and stock returns was positive (see Fig. 1). This is consistent with the model, which predicts that correlation is mostly positive when high-risk technology is relatively large (Fig. 5).

Table 4
Yield Curve and Bond-Stock Correlations. The table shows estimates of a regression of annual changes in the level and slope of the yield curve onto the changes in bond-stock correlations. Nominal treasuries in the first two columns, TIPS in the middle two columns, and model implied estimates in the last two columns are shown. t-statistics in parantheses use HAC standard errors with a small-sample correction.

	Data (nominal)		Data (real)		Model	
	ΔLevel	ΔSlope	ΔLevel	ΔSlope	ΔLevel	ΔSlope
ΔCorr	0.018	−0.013	0.014	−0.023	0.01	−0.002
	(4.81)	(−1.86)	(3.2)	(−0.21)	-	-
Obs.	11,738	11,738	4981	4981	-	-
R^2	0.12	0.05	0.18	0.002	0.51	0.22

Table 5
Fama-Bliss excess returns and yield changes regressions. The table shows coefficient estimates of a regression of one-year excess return on an n-year bond (left panel), and changes in a one-year yield n years from now (right panel) onto the n-year forward spread. Empirical estimates of the slope coefficient and their t-statistics, as well as the model-implied coefficients are provided for each regression. Rows correspond to different values of n from 2 to 5 years. t-statistics in parantheses use HAC standard errors with a small-sample correction.

	$rx_{t+1}^{(n)} = a_{rx} + b_{rx}\left(f_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+1}^{(n)}$			$y_{t+n-1}^{(1)} - y_t^{(1)} = a_y + b_y\left(f_t^{(n)} - y_t^{(1)}\right) + \epsilon_{t+1}^{(n)}$		
Maturity	Data		Model	Data		Model
(years)	$\hat{b}_{rx}$	t-stat	b_{rx}	$\hat{b}_y$	t-stat	b_y
2	0.70	2.14	0.60	0.30	0.91	0.40
3	0.86	2.05	0.72	0.71	1.60	0.28
4	1.01	2.14	0.87	1.03	3.23	0.16
5	1.16	2.29	0.46	1.06	4.09	0.45

4.3. The term structures of interest rates and risk premia

The model makes sharp predictions about dynamics of interest rates and term premia, and links them to the dynamics of bond-stock correlations. I now test these predictions.

4.3.1. The yield curve and bond-Stock correlations

Table 4 regresses annual changes in the level and slope of the yield curve onto the changes in bond-stock correlations. In the first two columns level, slope, and bond-stock correlation are constructed using nominal treasuries, while the next two columns focus on TIPS. The last two columns of the table provide estimates of the slope coefficients in the model. Level is defined as an average of yields at all maturities. Slope is defined as the difference between 15-year and 1-year bond yields.

The results are largely consistent across nominal and real data, and the model. Bond-Stock correlation has a strong positive level effect on the yield curve, that is, interest rates tend to increase in times when the bond-stock correlation increases. Bond-stock correlation also exhibits a mild negative effect on the slope of the yield curve, that is, yield curve is on average more upward-sloping when the correlation gets low. Table B.5 and Table B.6 in Supplementary Materials: Appendix show that the effect becomes stronger when the changes are computed at a 3-year horizon, as well as that results are robust for controlling for equity return volatility in the regressions.

4.3.2. Dynamics of term premia

In Table 5 I examine dynamics of term premia in the data and in the model. I estimate slope coefficients in Fama and Bliss (1987) regressions of one-year excess return on an n-year bond (left panel), and changes in a one-year yield n years from now (right panel), onto the n-year forward spread. Empirical estimates of the slope coefficient and their t-statistics, as well as the model-implied coefficients are provided for each regression. Empirical results are consistent with the prior evidence in the literature: at one-year horizon, forward spreads mostly forecast bond risk premia as opposed to yield changes — the expectations hypothesis is violated. We can observe similar patterns in the model.

4.4. Return dynamics in the cross-section of bonds and stocks

4.4.1. Dynamics of bond and stock risk premia

The model implies additional potential predictors of term and equity risk premia. The Bond-Stock correlation is one such predictor. The model makes two separate predictions. The first prediction—that bond (stock) risk premium should increase (decrease) with the *level* of bond-stock correlation—is difficult to test empirically due to extreme persistence and non-stationarity of bond-stock correlation as can be witnessed in Fig. 1. The second prediction of the model is that bond

Table 6
Dynamics of Bond and Stock risk premia. The table reports predictability of term premia on a 10-year treasury (r_b), equity index premia (r_s), and their difference ($r_s - r_b$) with annual differences in Bond-Stock correlation. The last three columns include controls for equity index volatility and log equity Price-Dividend ratio. t-statistics in parantheses use HAC standard errors with a small-sample correction.

	r_b	r_s	$r_s - r_b$	r_b	r_s	$r_s - r_b$
ΔCorr	0.06	−0.12	−0.19	0.09	−0.13	−0.22
	(1.87)	(−1.66)	(−3.33)	(2.58)	(−2.02)	(−3.76)
Vol	-	-	-	1.25	−4.87	−5.55
	-	-	-	(0.96)	(−2.36)	(−3.26)
pd	-	-	-	0.06	0.05	−0.01
	-	-	-	(2.19)	(1.15)	(−0.11)
Obs.	11,739	11,739	11,739	11,739	11,739	11,739
R^2	0.02	0.02	0.06	0.11	0.06	0.08

Table 7
Long-Short Equity Portfolio Predictability with Bond-Stock Correlation. Each column reports coefficient estimates and the R^2 of a regression of future one-year returns on a long-short portfolio onto the annual changes in Bond-Stock correlation. "H-L risk Industry" columns uses a long-short portfolio based on the definitions of low- and high-risk industries in Section 4.2.2. The remaining columns construct long-short portfolios based on stock characteristics. t-statistics in parantheses use HAC standard errors with a small-sample correction.

	H-L risk Industry	Size	Value	Profitability	Investment
ΔCorr	−0.07	0.01	0.14	−0.12	0.11
	(−1.51)	(0.05)	(1.79)	(−1.82)	(2.88)
Obs.	7604	11,007	11,007	11,007	11,007
R^2	0.02	0	0.03	0.03	0.05

(stock) risk premium should increase (decrease) with *changes* of bond-stock correlation. Fig. 6 provides some intuition for this relationship. In particular, bond and equity risk premia impulse responses show that bond risk premium raises and *stays elevated* for several years in response to an increase in bond-stock correlation. Likewise, stock risk premium falls and *stays depressed* for several years in response to an increase in bond-stock correlation. This means that sudden increases in correlation predict higher bond and lower stock risk premia in the future at multiple horizons. Because this prediction circumvents the issue of high persistence in bond-stock correlation, it might be a more powerful test of the model, as I indeed confirm empirically.

Table 6 shows that one-year changes in bond-stock correlation indeed forecast term premium in the data, but they also forecasts equity risk premium with an opposite sign. The table reports predictability of the term premia on a 10-year treasury (r_b), equity index premia (r_s), and their difference ($r_s - r_b$) with annual differences in Bond-Stock correlation. The last three columns control for equity index volatility and log equity price-dividend ratio.

The table shows that high Bond-Stock correlation predicts high term and low equity premium at an annual horizon.[8] While both are marginally significant, their spread can be forecasted especially strongly and significantly. To my knowledge this is a new empirical fact. It is consistent with the mechanisms and predictions of the model. Indeed, as can be seen from impulse response functions in Fig. 6, the model has strong predictions that bond and stock risk premia move in opposite directions in response to either of the two shocks in the model and stay elevated or depressed for a significant period of time.

4.4.2. Evidence from long-short equity portfolios

While a long term bond and the equity market index are perhaps the most natural empirical counterparts for the low- and high-risk technologies in the paper, predictions of the model, could, in principle, hold across different types of stocks if we assume that they predominantly employ either low- or high-risk capital. One such potential classification was discussed in Section 4.2.2, where I defined the share of high-risk capital x based on two groups of Fama-French industries: consumer non-durables, utilities, and shops as proxies for low-risk capital, and durables and manufacturing as high-risk.

In Table 7 I look at predictability of the long short portfolio underlying this definition of low- and high-risk capital, as well as a few other potential definitions based on stock-level characteristics commonly used in asset pricing: size, value, profitability, and investment. Each column reports coefficient estimates and the R-squared of a regression of future one-year returns on a long-short portfolio onto the annual changes in Bond-Stock correlation.

[8] I also document similar levels of predictability at all horizons up to five years in Appendix, section B.1.

Table 8
Covariance of Stock-Bond returns with changes in the risk-free and equity volatility. Top panel shows estimates of a regression of stock-bond returns or High-Low risk industry returns onto annual changes in the risk-free rate and/or equity volatility. Bottom panel shows model-implied estimates. *t*-statistics in parantheses use HAC standard errors with a small-sample correction.

	Δr_f	ΔVol	Obs.	R^2
		Data		
Stock-bond returns	5.29	−0.77	11,739	0.24
	(5.73)	(−3.97)		
	5.80	-	11,739	0.24
	(5.58)	-		
High-Low risk industry returns	2.35	−0.46	7854	0.12
	(3.12)	(−4.86)		
	3.03	-	7854	0.12
	(4.10)	-		
		Model		
	2.29	-	-	0.68
High-Low risk technologies	-	−0.35	-	0.70
	1.02	−0.20	-	0.71

The coefficient estimate for the long-short portfolio returns based on definitions of high- and low-risk industries from Section 4.2.2 shown in the first column of the table ("H-L risk Industry") is consistent with the estimates in Table 6: the coefficient is negative and close to being statistically significant at the 10% level. For value, profitability, and investment long-short portfolios, all coefficients are statistically significant at the 10% level (at 1% for investment), suggesting that these anomaly portfolios are predictable by the Bond-Stock correlation. Signs of the coefficients suggest that value firms, low profitability firms, and firms with low investment are more bond-like than the firms at the opposite ends of their corresponding long-short strategies. These firms tend to have more stable cashflows which are more consistently paid out to shareholders rather than reinvested. I also document similar levels of predictability for the "H-L risk Industry" at all horizons up to five years in Supplementary Materials: Appendix, section B.1.

In summary, I find bond-stock correlation to be a powerful predictor of the spread between bond and equity risk premia, as well as risk-premia of certain long-short portfolios. In either case, the two legs of the long-short strategy are empirical counterparts to risk premia on the two technologies in the model.

4.4.3. Covariance of equity and bond returns with financial variables

Beyond risk premia the model also implies certain testable relationships between observed contemporaneous quantities. In particular, Fig. 6, shows that marginal q of both technologies move in opposite ways in response to the two shocks in the model. The risk-free rate moves in the same and equity volatility in the opposite directions than the marginal q of the high-risk technology. This mechanism is consistent with empirical findings in Pflueger et al. (2020), who document positive correlation between detrended one-year real rates and the spread in book-to-market ratios between low- and high-volatility stocks.

Bond and stock realized returns respond to the risk aversion shock in the same way as their marginal q. The response of equities to the capital shock, however, is subdued or even goes in the opposite way due to a strong cashflow effect on their returns. This is the mechanism that induces co-movement of bond and stock returns in the model. In addition, because the risk-free rate and equity volatility vary disproportionally more in the short term in response to risk-aversion shocks, stock return should be negatively correlated with changes in equity volatility and positively correlated with changes in the risk-free rate.

I test these implications in Table 8. The top panel shows estimates of a regression of the long-short portfolio of stock minus bond returns (lines 1–2) or "High-Low risk industry" returns (lines 3–4) onto the annual changes in the risk-free rate and/or equity volatility. The bottom panel looks at the coefficient estimates implied by the model in two univariate regressions and a multivariate regression. The results show that coefficient estimates in the data are directionally consistent with the values implied by the model. Coefficients on the risk-free rate changes are positive and highly significant, suggesting that changes in the risk-free rate are more negatively correlated with bond returns (and low risk industries) than equity returns (high-risk industries). Coefficients on changes in volatility are negative and highly significant, indicating the opposite relationships. Both of these results are consistent with the impulse response functions in Fig. 6.

5. Conclusions

In this paper I develop a general equilibrium model capable of reconciling pervasive variation in bond term premia and correlation between real bond and stock returns, in both magnitude and sign. The model explains: (i) how bond and stock returns and conditional risk premia are linked: differences in exposure to the two sources of risk drive cross-sectional variation in returns in the model; (ii) why correlation of bond and stock returns changed signs in early 2000s: the model predicts correlation is mostly positive when high-risk technology is relatively large – consistent with empirical evidence on the relative sizes of high- and low-risk stock market sectors around 2000s; (iii) provides theoretical decomposition of bond term premia onto two components: compensations for the technology-diversification ("level") risk and the flight-to-safety ("discount-rate") hedging – consistent with empirical evidence in Kozak and Santosh (2020) that an ICAPM with news to discount rates as a factor, is able to jointly price cross-sections of bonds and equity portfolios.

The model delivers novel empirical predictions which I verify in the data: (i) Bond-Stock correlation co-moves with empirical measures of risk aversion such as volatility, VIX, and credit spreads; (ii) Bond-Stock correlation has a strong positive effect on the level and a mild negative effect on the slope of the yield curve; (iii) Bond-Stock correlation is a powerful predictor of bond and equity risk premia, and especially their long-short portfolio, as well as some equity-only long-short portfolios sorted on measures of their bond-likeness.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2021.12.004.

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